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Audited reputation ☆

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We study reputations with imperfect audit and a reputation market. The main result shows the existence of a separating equilibrium in the reputation market, which contrasts with Tadelis [Tadelis, S., 2002, The market for reputations as an incentive mechanism, Journal of Political Economy 110(4), 854–882].

1. Introduction

Reputation, as an economic concept to deal with moral hazard and/or adverse selection problems, has been studied by many researchers (Fudenberg and Levine, 1992; Mailath and Samuelson, 2001). In these studies, the principal's willingness to pay depends only on the agent's past performance as measured by reputation, and transaction outcomes are not contractible. This is typically justified by the fact that if the outcome were contractible, the principal and agents could simply assign a contingent contract to resolve any problems. In many scenarios in reality, however, the outcome can be verified or audited (at some cost). One such example is grid computing (Hidvegi and Whinston, 2006), where a principal can check computational results by repeating the computation process. When verification is not performed all the time due to the cost or imperfect audit, there is still room for moral hazard and adverse selection problems. This paper addresses the problems of both moral hazard and adverse selection, with imperfect audit.

We introduce an unbiased auditor to perform verifications on a random basis. Each agent operates a firm that has a reputation, and the reputation is updated according to the auditing result. Agents can trade firms together with reputations in the reputation market, and the shift of ownership is unobservable to consumers. Our major result shows the existence of a separating equilibrium, in which high-type agents buy firms with good reputation, and low-type agents buy firms with low reputation. Our result contrasts with that of Tadelis (2002), who assumes unverifiable outcome and reputation is based on consumer reports. Tadelis (2002) concludes that no separating equilibrium exists. We attribute this difference to the contingent payments we introduced, under which low-type agents have less incentive to mimic high types, since they may fail the audit and get no benefit from mimicking.

2. The model

We consider three entities: producers, consumers, and an auditor. We normalize the population of producers to 1, and producers can be one of two types, denoted by $\theta \in \{H, L\}$. The proportion of H-type producers is λ_H, and that of L-type producers is $1-\lambda_H$. There are two markets: the product market and the reputation market. Each product is associated with a probability of being successful. For simplicity, we assume reputation is the number of continuous successes j before a firm fails the audit. Firms' built-in reputations are separated from agents and can be traded on the reputation market. Both markets open for infinite periods, and in each period, the timeline of events can be described as follows.

First, producers trade reputations in the reputation market. The price for a reputation j is denoted by V_j. Producers have three choices: staying with the current reputation, selling that reputation and buying another one, or creating a new reputation. Second, each producer

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chooses an effort level $w\in[0,1]$ at cost $c(w)=k_\theta w^2$ to produce one product, where $\theta\in\{H,L\}$ and $k_L>k_H$. For simplicity, we assume w is also the probability of the product being successful. Each producer produces a batch of r products in each period, and within a batch, whether one product is successful is independent of another. The quality of a certain product is unobservable to consumers, but is verifiable by the auditor. Next, the auditor randomly picks one product from each batch to check.[1] If it turns out to be successful, the producer's reputation is adjusted from j to $j+1$ and it is allowed to sell the batch of products; if it turns out to be a failure, the producer's reputation is changed to 0 and the batch of products is taken away by the auditor.[2] The random audit follows from the well-established audit sampling procedure (Rittenberg and Schwieger, 2005). A successful product has value 1 to a consumer, and a failed product has value 0 to a consumer. We assume that consumers are risk neutral and have no bargaining power, so they pay their expected value of products. Thus, given $\tilde{w}$ is their belief about a product being successful, the price for this product is $\tilde{w}$.

We assume that creating a new reputation is free (i.e., $V_0=0$), as a normalization. We also assume that shifts in reputation ownership are unobservable to consumers. This indicates the separation of producers from reputations. In reality, to observe shifts of owners is usually very costly, especially to individual consumers. Technically, we let $2k_\theta-1>0$. This means the marginal cost of production is higher than the marginal revenue when the highest effort ($w=1$) is exerted. Otherwise, it is possible that exerting the highest effort is agents' best choice, and thus the model will become trivial.

3. Existence of a separating equilibrium

A separating equilibrium consists of a set of values $\{w_j,V_j\}$, $j=0,1,\ldots$ and a cutoff reputation h: any reputation $j<h$ is chosen by low-type producers only and any $j\geq h$ is chosen by high-type producers only. Denote $\tilde{w}_j$ as the market belief of the effort level by producers with reputation j, while w_j is the equilibrium effort level chosen by the producer. The expected payoff from producing r units is $[wr\tilde{w}-rk_\theta w^2+(wV_{j+1}-V_j)]$. The first term is the product of price and the expected probability of passing the audit; the second term is simply the cost of effort; and the third term is the expected change in reputation value. We simply consider the expected payoff per unit product, and define π_j^θ as the maximum expected payoff per unit for a type θ producer under reputation j. We then write π_j^θ in the following form:

$$\pi_j^\theta=\max_w w\tilde{w}_j-k_\theta w^2+\frac{wV_{j+1}-V_j}{r}. \tag{1}$$

In the separating equilibrium described above, all low-type producers must be indifferent to holding any reputation $j<h$. Otherwise, reputation prices would have to change to balance the market. Similarly, all high-type producers are indifferent to holding reputations $j\geq h$. Denote equilibrium profits for low and high types as π_L and π_H, respectively. For now, we use π_L and π_H as given, and we will show later how π_L and π_H can be calculated.

3.1. Low-type producers' effort and payoff

In a separating equilibrium, low-type producers' optimal effort level, termed w_j, is determined by the first-order condition of the objective function (1):

$$\tilde{w}_j-2k_Lw_j+\frac{V_{j+1}}{r}=0,\ \text{for } j=0,1,\ldots h-1. \tag{2}$$

Notice that in equilibrium, the market's belief about effort levels is consistent with producers' equilibrium effort levels, i.e., $\tilde{w}_j=w_j$. Based on this, we can solve for the equilibrium effort levels as:

$$w_j=\frac{V_{j+1}}{(2k_L-1)r}. \tag{3}$$

Furthermore, we can derive the equilibrium profit π_L by substituting the first-order condition (2) into the payoff function (1):

$$\pi_L=k_Lw_j^2-\frac{V_j}{r}. \tag{4}$$

Notice that $V_0=0$. Provided the equilibrium profit is π_L, we can derive w_0 by Eq. (4), V_1 by Eq. (3), w_1 by Eq. (4), and so on, until we get all w_j, $j=1,2,\ldots,h-1$, and all V_j, $j=1,2,\ldots,h$.

Lemma 1. *In a separating equilibrium, both reputation prices V_j and effort levels w_j increase in reputation j.*

This lemma can be inferred from Eqs. (3) and (4). First, we have $V_1>V_0=0$. By Eq. (4), if $V_{j+1}>V_j$, then $w_{j+1}>w_j$, which in turn indicates $V_{j+2}>V_{j+1}$ by Eq. (3). Thus, $V_{j+1}>V_j$ and $w_{j+1}>w_j$ for $j=0,1,\ldots,h-1$. This result means that in a separating equilibrium, low-type producers are induced to work harder as the reputation goes higher in order to lower the probability of being caught and losing the value of reputation.[3]

3.2. High-type producers' effort and payoff

To capture a plausible equilibrium, we assume that in a separating equilibrium all reputations $j\geq h$ have the same value V_h, and all high-type producers exert the same level of effort w_h. One justification is that producers' types can be inferred from reputations, and there is no need to further differentiate among high-type producers. Applying an analysis similar to that for low-type producers, we can derive high types' equilibrium effort level and payoff as:

$$w_h=\frac{V_h}{(2k_H-1)r}, \tag{5}$$

$$\pi_H=k_Hw_h^2-\frac{V_h}{r}. \tag{6}$$

Notice that V_h can be obtained from Eq. (3): $V_h=r(2k_L-1)w_{h-1}$.

3.3. Determining π_L

From the above analysis, all other variables (i.e., w_j, V_j, and π_H) can be determined in equilibrium as functions of π_L. In this subsection, we show that the composition (λ_H) of the producer population determines π_L. Denote λ_j as the proportion of reputation j. We study a steady state case where proportions of reputations λ_j, $j=1,\ldots,h,\ldots$ remain the same across periods, despite the dynamics of the producer population.

$$1-\lambda_H=\sum_{j=0}^{h-1}\lambda_j=\lambda_0+\sum_{j=1}^{h-1}\lambda_0\prod_{i=0}^{j-1}w_i, \tag{7}$$

$$\lambda_H=\sum_{j=h}^{+\infty}\lambda_j=\sum_{j=h}^{+\infty}\lambda_0\prod_{i=0}^{j-1}w_i. \tag{8}$$

The first equation means that in a separating equilibrium the total proportion of reputations below h equals the proportion of low-type

[1] Notice that, since the unit chosen is independent from the rest in the batch, the audit is imperfect in the sense that the quality of the remaining products is not reflected in the audit result.

[2] The purpose of taking away failed batches is to punish producers for being caught by the audit. Since these batches contain successful products, the auditor can still sell them to consumers to avoid efficiency loss.

[3] Notice that the monotonicity here is certainly related to the punishment scheme in our reputation system: once a product fails, an agent's reputation goes to zero. The monotonicity may be absent for general reputation systems.

producers. The second equation is for high-type producers and has a similar interpretation. From the above two equations, we have[4]

$$\lambda_H = \frac{\prod_{i=0}^{h-1} w_i}{(1-w_h)\sum_{j=0}^{+\infty}\prod_{i=0}^{j-1} w_i}. \tag{9}$$

Notice that w_i's are functions of π_L. We can show there exists a unique π_L satisfying Eq. (9), so we can express π_L as $\pi_L(\lambda_H, k_L, k_H)$. (The existence and uniqueness of π_L are presented in the Appendix.)

3.4. Existence of a separating equilibrium

To show that the separating equilibrium does exist, we need to prove that both low-type and high-type producers have no incentive to deviate, i.e., we need to check incentive compatibility (IC) conditions for both types.

(i) IC for low-type producers: To mimic high types, low-type producers maximize Eq. (1) under $\tilde{w}_h = w_h$. Based on the first order condition, we get $w_h^L = \frac{w_h + \frac{V_h}{r}}{2k_L} = \frac{k_H}{k_L} w_h$, and $\pi_h^L = k_L\left(w_h^L\right)^2 - \frac{V_h}{r}$. The incentive compatibility requires $\pi_h^L \le \pi_L$.

(ii) IC for high-type producers: Similarly, we get $w_j^H = \frac{w_j + \frac{V_{j+1}}{r}}{2k_H} = \frac{k_L}{k_H} w_j$ and $\pi_j^H = k_H\left(w_j^H\right)^2 - \frac{V_j}{r}$. Notice that $\pi_j^H = k_H\left(\frac{k_L}{k_H} w_j\right)^2 - k_L w_j^2 + \pi_L$, which is increasing in j since w_j increases in j and $k_H\left(\frac{k_L}{k_H} w\right)^2 - k_L w^2 + \pi_L$ increases in w. This means π_{h-1}^H is the highest profit that a high type can get by mimicking a low type. Therefore, it is sufficient to show $\pi_{h-1H}^H \le \pi_H$.

Proposition 2. (Existence of a separating equilibrium) *A separating equilibrium exists if* $0 < \pi_L(\lambda_H, k_L, k_H) < 1 - k_H - \frac{k_L(k_L-k_H)}{k_H}\left(\frac{2k_H-1}{2k_L-1}\right)^2$.

Intuitively, this condition means that as long as low-type producers' profit is low enough, high-type producers will not be interested in pooling with them. On the other hand, this profit should be no less than zero so that low-type producers are willing to participate and to stay with low reputations.

Proposition 3. *In a separating equilibrium, the equilibrium profit for low-type producers π_L increases in the proportion of high-type producers λ_H.*

This is because a higher λ_H means a higher demand but a lower supply of reputation h, which leads to a higher V_h. Thus, a low-type producer who luckily reaches h will be paid higher, which indicates a higher π_L.

4. Conclusion and discussion

In this paper, we targeted the category of verifiable products, and showed the existence of a separating equilibrium in a reputation market. We introduced an auditor to check on outcomes and to implement contingent payments to agents based on the auditing results. By combining incentives from both reputation and audit, we are able to derive the separating equilibrium, which is lacking with non-verifiable products (Tadelis, 2002). A separation has an important interpretation in the online environment, where people face a high risk of manipulation and fraud.

The separating result derived is robust to different reputation measures and auditing procedures.[5] For example, if we consider the proportion of success for the last T samples (1 in each period) as the reputation measure, it can be shown that a similar separating equilibrium exists.[6] In terms of auditing procedures, we can relax the current strict rule by, for example, randomly auditing two products from r, and requiring at least one success for payments and reputation to increase. We can also show that a similar separation exists.

As in many signaling games, however, multiple equilibria may arise. In that case, certain refinements can be applied to rule out some of the equilibria (e.g., *intuitive criterion*, proposed by Cho and Kreps, 1987). On the other hand, it is worth pointing out that, for a given h, our analysis generates a unique separating equilibrium, if any, although there may exist multiple h's that lead to separation.

Appendix A

Determining π_L

We divide the denominator of the right-hand side (RHS) of Eq. (9) by $\prod_{i=0}^{h-1} w_i$, and rearrange it into:

$$\lambda_H = \frac{1}{1 + (1-w_H)\left[\frac{1}{\prod_{i=0}^{h-1} w_i} + \frac{1}{\prod_{i=1}^{h-1} w_i} + \ldots + \frac{1}{\prod_{i=h-2}^{h-1} w_i}\right]}.$$

It's easy to see that the RHS is increasing in $w_i, i = 0,1,\ldots,h$. Also, as π_L increases, w_0 increases according to Eq. (4), and thus V_1 increases according to Eq. (3); and again w_1 increases due to Eq. (4), and so on, such that all w_j increase. Thus, RHS is increasing in π_L, or π_L is increasing in λ_H.

On the other hand, when w_h is close to 1, the denominator goes to 1, and thus RHS becomes 1. When any w_j is close to 0, the denominator becomes infinitely large, and thus, the RHS is 0. Therefore, as π_L varies, RHS varies from 0 to 1. Combining with the monotonicity of RHS, given any $\lambda_H \in [0, 1]$, there exists a unique π_L.

Proof of existence

Denote $A \equiv k_L\left(\frac{k_H}{k_L}\frac{2k_L-1}{2k_H-1}\right)^2$ and $B \equiv \left[k_H\left(\frac{2k_L-1}{2k_H-1}\right)^2 - \frac{k_L(k_L-k_H)}{k_H}\right]$. It is easy to verify that $B > A > 0$.

The IC condition $\pi_{h-1}^H \le \pi_H$ can be rearranged into

$$\left[k_H\left(\frac{2k_L-1}{2k_H-1}\right)^2 - \frac{k_L(k_L-k_H)}{k_H}\right] w_{h-1}^2 - (2k_L-1)w_{h-1} - \pi_L \ge 0,$$

which requires $w_{h-1} \ge \frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B}$ (note $w_{h-1} \ge 0$). Notice that because $w_h \le 1$ and $w_h = \frac{2k_L-1}{2k_H-1} w_{h-1}$, we must have $w_{h-1} < \frac{2k_H-1}{2k_L-1}$. Therefore, the above IC condition is equivalent to

$$w_{h-1} \in \left[\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B}, \frac{2k_H-1}{2k_L-1}\right]. \tag{A.1}$$

Similarly, solving the other IC condition $\pi_h^L < \pi_L$, we get

$$w_{h-1} \in \left[0, \frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4A\pi_L}}{2A}\right]. \tag{A.2}$$

A separating equilibrium is supported if w_{h-1} satisfies both Eqs. (A.1) and (A.2). Notice that $\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B} < \frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4A\pi_L}}{2A}$ because $B > A$ and $\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4x\pi_L}}{2x}$ decreases in x (which can be verified by checking the first-order derivative). Therefore, as long as

$$\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B} < \frac{2k_H-1}{2k_L-1}, \tag{A.3}$$

there exists w_{h-1} satisfying both Eqs. (A.1) and (A.2). By simple algebra, Eq. (A.3) reduces to $\pi_L < \left(\frac{2k_H-1}{2k_L-1}\right)^2 B - (2k_H-1)$, which is equivalent to (by substituting in B)

$$1 - k_H - (k_L-k_H)\frac{k_L}{k_H}\left(\frac{2k_H-1}{2k_L-1}\right)^2 - \pi_L > 0. \tag{A.4}$$

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[4] We define $w_{-1} = 1$ in order to better arrange the equation.

[5] We thank the reviewer for comments on the robustness of the model.

[6] Notice that the equilibrium in the special case $T = 1$ is equivalent to the equilibrium of $h = 1$ in the current reputation measure.

Given the range for w_{h-1}, we can determine the cutoff value of h on a trial basis. (a) Let $h=i$ ($i \geq 0$). (b) Calculate $\{w_j, V_j\}$ and π_L. (c) Check if Eq. (A.4) is satisfied. If yes, we get the equilibrium; otherwise, try $h=i+1$, and repeat (a)~(c).

References

Cho, I.-K., Kreps, D.M., 1987. Signaling games and stable equilibria. Quarterly Journal of Economics 102 (2), 179–221.

Fudenberg, D., Levine, D.K., 1992. Maintaining a reputation when strategies are imperfectly observed. Review of Economic Studies 59 (3), 561–579.

Hidvegi, Z., Whinston, A.B., 2006. Mechanism design for grid computing, Working paper.

Mailath, G.J., Samuelson, L., 2001. Who wants a good reputation? Review of Economic Studies 68 (2), 415–441.

Rittenberg, L.E., Schwieger, B.J., 2005. Auditing: Concepts for a changing environment, Chapter 9 (South-Western), 6th ed., pp. 334–368.

Tadelis, S., 2002. The market for reputations as an incentive mechanism. Journal of Political Economy 110 (4), 854–882.